**Conditional Probability Distribution**

A **Conditional Probability Distribution** describes the probability of an event occurring given that another event has already occurred. It is fundamental in probability theory and statistics, especially in Bayesian inference and machine learning.

### ****Mathematical Definition****

The conditional probability of an event A given event B is:

P(A | B) = P(A ∩ B) / P(B), provided that P(B) > 0.

For **discrete random variables** X and Y, the conditional probability mass function (PMF) is:

P(X = x | Y = y) = P(X = x, Y = y) / P(Y = y).

For **continuous random variables**, the conditional probability density function (PDF) is:

f(X | Y) = f(X, Y) / f(Y),

where f(X, Y) is the joint probability density function, and f(Y) is the marginal probability density function of Y.

### ****Example (Discrete Case)****

Suppose we have a biased die where:

P(X=1) = 0.1, P(X=2) = 0.2, P(X=3) = 0.3, P(X=4) = 0.4.

If we know that X is at most 3 (Y), we can compute:

P(X = 2 | X ≤ 3) = P(X=2) / P(X ≤ 3) = 0.2 / (0.1 + 0.2 + 0.3) = 1/3.

### ****Example (Continuous Case)****

If X and Y are jointly normal with means μX, μY, variances σX^2, σY^2, and correlation ρ, the conditional distribution of X given Y = y follows:

X | Y = y ~ N(μX + ρ (σX / σY) (y - μY), (1-ρ^2) σX^2).

### ****Applications****

* **Bayesian Inference**: Used in Bayesian statistics to update beliefs based on new evidence.
* **Markov Models**: Conditional probability distributions are key in Hidden Markov Models (HMMs).
* **Machine Learning**: Naïve Bayes classifier relies on conditional probabilities.